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A High Froude Number Time-Domain Strip Theory for Ship Motion Predictions in Irregular Waves

B. French[1], G. Thomas[1], M. Davis[2] and D. Holloway[2]

[1]National Centre for Maritime Engineering and Hydrodynamics
Australian Maritime College, University of Tasmania, Launceston, Tasmania 7248, Australia

[2]School of Engineering
University of Tasmania, Hobart, Tasmania 7005, Australia

Abstract

The prediction of ship motion characteristics early in the design stage in realistic sea conditions are of vital importance for the ship designer. Strip theories are commonly used for this purpose as they are fast and inexpensive. In this paper, an existing two-dimensional time-domain strip theory optimised for multi-hull vessels travelling at high Froude numbers is extended to predict motions in an irregular seaway. The encountered wave environment is represented by the superposition of regular sinusoidal waves. A method for decomposing idealised sea spectra into component regular waves of varying frequencies and constant amplitudes is presented. Ship motion predictions in irregular waves are then verified by conducting a spectral analysis on the motions and wave environment and comparing with motion predictions in regular waves. A method of ensemble averaging of spectra over a series of runs is adopted to reduce spectral variance. The extended seakeeping method is then validated by comparing predicted motions of a large high-speed catamaran in irregular seas with scale model results from towing tank experiments.

Introduction

Accurate prediction of the motion characteristics of a vessel early in the design spiral is vital for a successful design. Potential flow strip theory methods are commonly used for this purpose as they are fast and inexpensive in computational requirements. This paper presents an irregular wave extension to a time-domain method, giving ship designers the ability to predict vessel motions in realistic irregular seas numerically.

A two-dimensional time-domain strip theory optimised for high Froude numbers is used to predict vessel motions. The algorithm was originally developed to predict motions in regular waves, and then extended to predict global wave loads. The work presented here is a further expansion of this method to model motions in irregular seas.

There are many motion prediction methods available to the ship designer, each with the same goal but varying in approach. However, all motion prediction methods must simplify the problem by making limiting assumptions, and this is particularly evident when predicting motions of high-speed catamarans.

The seakeeping theory described above forms the basis of the motion and global load prediction tool and has been extended for irregular wave applications. The irregular wave version of the time-domain seakeeping method optimised for high Froude numbers is verified in this work by performing a series of program tests and validated by comparing scale model experiment results from towing tank tests with the predicted motions and wave spectra produced by the seakeeping program.

Time-Domain Strip Theory Description

A two-dimensional time-domain strip theory capable of predicting motions and global wave loads of large high-speed catamarans in regular waves has been developed at the University of Tasmania [1]. It is based on the transient Green function solution for strips of water which are fixed in space and perpendicular to the direction of motion. The solution for each strip starts when the bow enters the strip, and finishes when the stern leaves the strip. The Green function used satisfies the linearised free surface boundary condition; therefore if the water depth is considered to be deep, it is only necessary to place sources on the hull surface. This has the advantage of reducing the number of sources (and thus number of computations) required for a solution [1].

Large amplitude motions and irregular incident waves can be simulated realistically with this model because the hull is panelled up to the instantaneous incident wave free surface at each time step. This requires the sources to be redistributed on the wetted hull surface at each time step. It is important to remember that the Green function linearises the free surface boundary condition, so any non-linear effects resulting from large motions of the free surface are not modelled. The Green function solution determines the local pressures on each hull surface panel and the total force on the hull is found by integrating over the hull surface at each time step. The hull is then treated as a rigid body and instantaneous accelerations in heave, pitch, roll, yaw and sway are determined. It is then possible to integrate the accelerations to determine the motion of the vessel through time [2].

Davis and Holloway [3] compiled a comprehensive study validating this code for 14 different hull forms against towing tank data for conventional slender hulls suitable for high speed catamarans, monohulls, SWATH (small water area twin hull) and semi-SWATH hull forms. Motions of an NPL 5b catamaran model were also satisfactorily predicted in oblique seas in a separate investigation [4]. Peak magnitudes of the response amplitude operators (RAOs) were found to be particularly sensitive to damping effects. Therefore the empirical vertical damping force per section equation was introduced:

$$D = \frac{1}{2} C_s \rho U v B \qquad (1)$$

where D is the vertical force per unit length, C_s is the vertical damping force coefficient, ρ is density, B is the sectional beam, U is the forward speed of the ship and v is the vertical velocity of the section relative to local water surface.

Increase of this damping coefficient can significantly reduce the peak responses of the heave RAO, with smaller reductions in pitch RAO. However, it does little to shift the resonant peak in frequency. The values for C_s vary, but are less than 0.15 and commonly of the order of 10^{-2} for simple geometries [3].

Prediction of Motions in Irregular Seas

In order to simulate realistic seakeeping scenarios, the method should be able to predict motions in an irregular seaway. Because this method is formulated in the time-domain and instantaneous wave heights are determined at all strips of water and at each time step, it is possible to use the principle of linear superposition of regular waves to create an irregular wave field. This assumes that nonlinear interactions of regular waves are negligible. An array of wave heights, frequencies, phases and headings are required in this method. Surface displacements, velocities and Froude-Krylov forces are calculated for each regular wave component at each water section and then summed to give the total surface displacement, velocity and Froude-Krylov forces.

Incident Irregular Wave Definition

The potential function for the j^{th} regular wave component is:

$$\phi_j = \frac{H_{wj}\omega_{0j}}{2k_{0j}} e^{k_{0j}z} \cos(k_{0j}x + \omega_{0j}t) \tag{2}$$

H_w is the wave height, ω_0 and k_0 are the angular wave frequency and wave number respectively, t is time and x and z are spatial distances. Linear superposition of individual wave components is used to obtain the total potential function for the irregular wave:

$$\phi = \sum_{j=1}^{n} \phi_j \tag{3}$$

for n regular waves.

The wave elevation associated with the j^{th} wave component is:

$$\eta_j = \frac{H_{wj}}{2} \sin(k_{0j}x + \omega_{0j}t). \tag{4}$$

Total wave elevation is simply the sum of these components ($\eta = \sum \eta_j$).

The hydrodynamic component of pressure head and vertical velocity can now be determined:

$$\frac{p}{\rho g} + z = -\frac{1}{g}\frac{\partial \phi}{\partial t} \tag{5}$$

Assuming deep water (depth > 0.5 wave length), the right hand side of equation (5) can be shown to be:

$$\frac{p}{\rho g} + z = \sum_{j=1}^{n} \eta_j e^{k_{0j}z} \tag{6}$$

where η is the wave elevation. The component of vertical velocity, w, is defined as the change in potential over the change in vertical distance ($w = \frac{\partial \phi}{\partial z}$) and is shown by equation (7):

$$w = \sum_{j=1}^{n} \frac{\partial \eta_j}{\partial t} e^{k_{0j}z} \tag{7}$$

Equations (2) through to (7) assume that the waves are travelling in the $-x$ direction. However these equations can be further generalised to represent any heading by replacing x by $(x \cos\theta + y \sin\theta)$. Each wave component can also be given a different phase by simply including phase angle γ in the $(k_{0x} + \omega_{0t})$ term.

Simulating Idealised Wave Energy Spectra

Any wave spectrum can be represented by a series of regular waves of varying frequency and amplitude depending on the energy distribution of the wave spectrum. The average energy $(\bar{E})$ over a wavelength is given by equation (8):

$$\bar{E} = \frac{\rho g \zeta_0^2}{2} \tag{8}$$

where ζ is defined as the wave amplitude. Figure 1 shows a wave energy spectrum that has been divided into a number of bands. The solid lines represent the boundaries of the frequency bands and the dotted lines show the mid-point, each band is represented by one regular wave. The n^{th} regular wave component of the spectrum can be found by applying equation (9), where the frequency of the regular wave is the mid-point and its amplitude represents the average energy over the frequency band.

$$\zeta_n = \sqrt{\frac{2\bar{E}}{\rho g}} \tag{9}$$

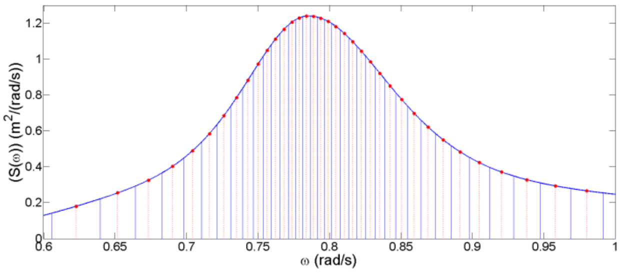

Figure 1: A JONSWAP wave spectrum illustrating the constant energy method of separating the spectrum into a series of regular waves. The dashed lines and points show the mid-point of each energy band, a regular wave of this frequency represents the energy within the associated band ($H_{1/3}$ = 2.24m, T_0 = 8s).

In order to avoid repetition of the wave pattern in time, the wave spectrum is not divided by a constant frequency but rather by constant amplitude. Each wave component represents a constant amount of energy in the spectrum; therefore in regions of little energy, the frequency spacing between wave components is greater than at the energy peak where the regular wave components are more concentrated. This can be seen in Figure 1, each frequency band represents the same amount of energy (and thus area under the curve). The spectrum shown in the above figure divided into fifty waves for clarity, in the simulations presented here, the wave spectra are divided into five hundred regular wave components.

Ship Motions in Irregular Waves

Response amplitude operators are determined for heave and pitch as a means of verifying the method. In the case of regular waves, the maximum motion response is divided by the wave height (or slope in the case of pitch) to determine the dimensionless response amplitude for the given wave frequency. In the more complicated case of motions in a wave spectrum, this is done by calculating the encountered wave spectrum from time domain measurements of the encountered wave elevation (or wave slope) and the motion spectrum from the response of the vessel, via a Fourier analysis. The discrete Fourier transform is used to determine the motion and wave spectra and the ratio of these spectra provides the RAO for that particular condition.

The motion RAO for heave is given as the square root of the ratio of the heave spectrum, $S_{x_{30}}(\omega_e)$, to the measured encountered wave elevation spectrum, $S_\zeta(\omega_e)$ and similarly the pitch RAO is given as the square root of the ratio of the encountered pitch spectrum, $S_{x_{50}}(\omega_e)$, to the wave slope spectrum, $S_\alpha(\omega_e)$:

$$\frac{x_{30}}{\zeta_0} = \sqrt{\frac{S_{x_{30}}(\omega_e)}{S_\zeta(\omega_e)}} \quad (10)$$

$$\frac{x_{50}}{k\zeta_0} = \sqrt{\frac{S_{x_{50}}(\omega_e)}{S_\alpha(\omega_e)}} \quad (11)$$

Here, x_{30} and x_{50} are heave and pitch motions respectively. Variances in the motions and wave spectra were reduced by applying Bartlett's method of ensemble averaging, as shown by equation (12). Instead of separating the runs into K segments before applying the Fourier analysis, each run was considered individually and the resulting spectra for all K runs in the condition were averaged. Bartlett's method was also applied to the time domain simulation data; a series of K = 20 runs were conducted and ensemble averaging smoothed the resulting spectrum.

$$S = \frac{1}{K}\sum_{i=1}^{K} S_i \quad (12)$$

To ensure that the same wave train was not encountered twice, a random phase was applied to each regular wave component when constructing the irregular was spectrum in the time domain, before it was input into the seakeeping program.

Verification

A series of system tests were devised to verify the operation of the extended seakeeping code. These tests ranged from simple checks such as observing motions in extremely long waves and comparing motions in a single regular wave with that of the same wave composed of two wave components of identical frequency to developing an operating strategy regarding simulating idealised sea spectra that make up the irregular spectrum.

Comparisons between motion response in regular waves and irregular waves were examined as a verification exercise. The motion response of the vessel in an irregular wave spectrum cannot be directly compared to regular sea results, due to variances in the significant wave heights and frequencies of the encountered waves in the spectrum (particularly if the vessel hull form results in nonlinear motions at larger wave heights). However the vessel response should fall within the ‘envelope’ of regular wave RAOs for different wave heights.

Figure 2 shows the predicted RAOs for a 112m high-speed wave-piercing catamaran sailing at 38kts in regular waves of varying frequency and wave heights with the motions in an irregular JONSWAP spectrum ($H_{1/3}$ = 2m, T_0 = 10s). The encounter frequency, shown as the abscissa in Figure 2, has been made non-dimensional by multiplying the encounter frequency by the square root of the ship length, L, divided by the gravitational constant, g, as shown by equation (13). Frictional effects such as viscosity have been neglected in these results.

$$\omega_e^* = \omega_e\sqrt{L/g} \quad (13)$$

The time-domain method is capable of capturing nonlinear behaviour as seen in the RAOs from regular waves, with decay in non-dimensional response (heave or pitch) observed around the peak motion response with increasing wave height. The motion response of the vessel in a 2m JONSWAP spectrum falls within the envelope of regular wave RAOs for wave heights of 1 and 2m, providing confidence in the irregular wave methodology.

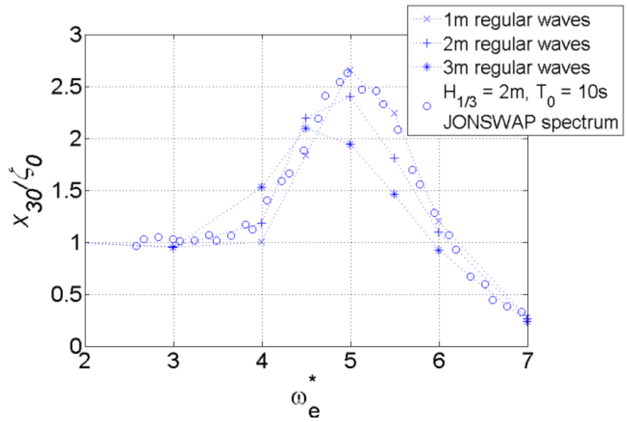

Figure 2: Numerical heave RAOs for a 112m wave-piercing catamaran. 1m, 2m and 3m regular waves (U = 38kts) compared with the response from a JONSWAP wave spectrum ($H_{1/3}$ = 2.0m, T_0 = 10s, U = 38kts).

Validation

Validation of the seakeeping code is primarily carried out by comparing predicted motion from the seakeeping code against scale model towing tank test results. In place of attempting to recreate the scale model test environment exactly in the computational domain (i.e. match time histories of wave elevations with the scale model experiments) a statistical approach was taken and a series of wave systems representative of each condition tested was developed using the method outlined previously. Twenty of these systems were constructed and ship motions were predicted using the irregular wave seakeeping method for each one. A Fourier analysis on wave elevation and ship motion time traces were conducted using Bartlett's method of ensemble averaging over the twenty runs representing each test condition. The resulting encountered wave elevation and slope spectra were compared with those measured experimentally from the scale model testing. Predicted heave and pitch spectra were divided by the respective wave elevation or slope spectra to obtain RAOs and these were also compared with experimental results.

A 2.5m scale model of a 112m high-speed wave-piercing catamaran was tested in irregular waves in a towing tank to validate the irregular seakeeping code. This catamaran contains a centre bow: a small third hull located between the demihulls on the bow of the vessel to provide extra buoyancy in the event of excessive motions of the bow. This bow was modelled as a short third hull in the numerical method.

Relatively mild sea conditions were selected in order to minimise non-linearties associated with large motions, such as wetdeck slamming. A JONSWAP wave spectrum was chosen with a significant wave height of 1.12m (25mm model scale) and modal period of 8s (1.2s model scale).

Encountered wave elevations and slope spectra from the model experiments and the seakeeping prediction code were compared with the ideal spectra to ensure that a viable comparison could be made between experimental and numerically predicted motions. Figure 3 shows the encountered wave elevation spectrum for the tested condition. The dot-dashed line is the spectrum produced from the seakeeping prediction while the dashed line represents the spectrum measured during experiments. Good correlation can be seen between the ideal and measured spectra, allowing viable comparisons between predicted and experimentally measured ship motions.

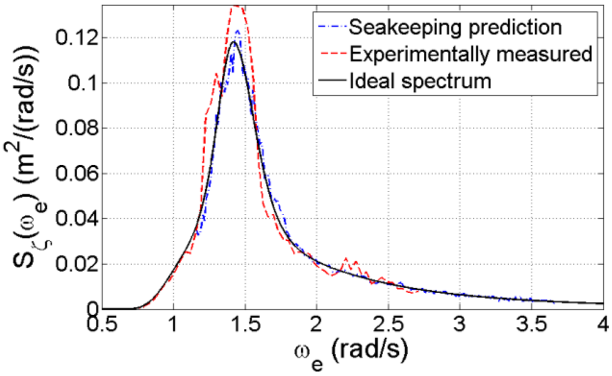

Figure 3: Encountered wave spectra (numerical and measured) compared with the ideal for a JONSWAP spectrum ($H_{1/3}$ = 1.12m, T_0 = 8s, U = 38kts, (25mm, 1.2s, 2.98m/s model scale)).

Figure 4 and Figure 5 compare the heave and pitch RAOs respectively. Since strip theories inherently ignore viscous effects, a vertical damping force coefficient of 0.08 is introduced to model vertical damping (see equation (1)). The peak heave RAO magnitude matches well with experimental results. However, there is a discrepancy between the predicted and measured motion frequency, both predicted heave and pitch RAOs appear to be 'shifted' to the right by approximately an increment of 0.8 in ω_e^*. This has also been observed when comparing full scale wave-piercing catamaran motions data with the simulation [5]. A number of possible causes of this mismatch were identified, ranging from the action of the centre bow to the added mass calculations at the transom stern. The irregular wave analysis method was not found to be the cause of this problem as the frequency offset in peak motions was also present in regular wave solutions. However, the frequency offset is not prevalent for simple hull forms (such as the NPL hulls presented by Davis and Holloway [3]), suggesting that some features of the more complicated wave-piercing catamaran hull form are not being modelled correctly in the simulation. In particular the potential flow solution does not represent separation at hard chines in the wave-piercing catamaran hull form near the stern which are likely to increase sectional added water mass effects.

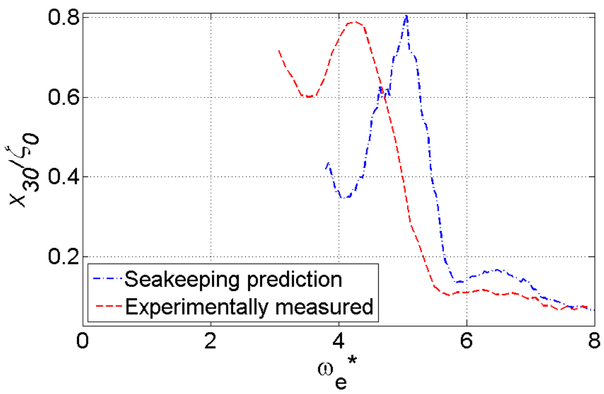

Figure 4: Comparison between experimental and predicted heave RAOs ($H_{1/3}$ = 1.12m, T_0 = 8s, U = 38kts, (25mm, 1.2s, 2.98m/s model scale)).

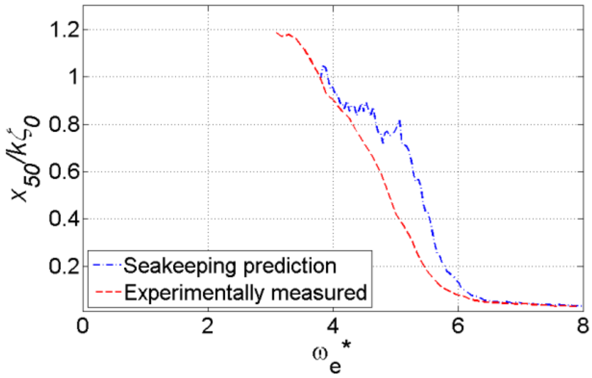

Figure 5: Comparison between experimental and predicted pitch RAOs ($H_{1/3}$ = 1.12m, T_0 = 8s, U = 38kts, (25mm, 1.2s, 2.98m/s model scale)).

Conclusions

A time-domain strip theory optimised for high speed multi-hull seakeeping was extended for the purpose of predicting motions and loads in an irregular seaway. A method for representing idealised wave spectra as a series of regular sinusoidal waves with identical amplitudes, random phases, and frequencies dependent on the energy distribution of the wave spectrum was developed as an input to the extended seakeeping method.

The method was verified by conducting a series of tests to ensure that the extension to the existing code was successful and then the method was partially validated by comparing seakeeping predictions with scale model experiments in a JONSWAP spectrum. Whilst the method is capable of modelling the non-linear motion response with changing significant wave height, a persistent offset in peak response frequency is present and requires further investigation, in particular the effect of hard chines in the ship sections.

Acknowledgments

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